Algebra and solid Exam

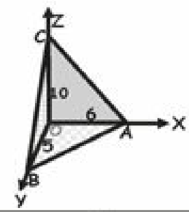
Answer the following questions 20 questions

From 1to 12 choose the correct answer

Q(1)If the plane $10X + 12Y + 6Z = 60$ intersected with the axes X,Y and Z at the points A ,B and C respectively then the volume of the solid ABCO where O is the origin point equals .....cube unit

① 20 ② 30

③ 50 ④ other wises



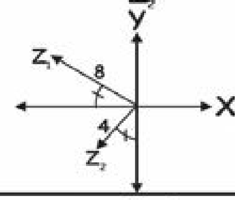
Volume of pyramid $= \frac{1}{3}$base area × height

$= \frac{1}{3}\left(\frac{1}{2} \times 5 \times 6\right) \times 10 = 50$

Q(2) In the opposite figure: Z_1, Z_2 are two complex numbers then $\frac{Z_1}{Z_2} =$

① 2 ② 2i

③ -2 ④ -2i



$\frac{8(\cos 180^\circ - \theta + i\sin 180^\circ - \theta)}{4(\cos -90^\circ - \theta + i\sin -90^\circ - \theta)} = 2(\cos 270^\circ + i\sin 270^\circ) = -2i$

Q(3) If Z_1, Z_2 are two complex numbers the amplitude of $(Z_1 Z_2) = \frac{5\pi}{18}$

And the amplitude of $\left(\frac{Z_1}{Z_2}\right) = \frac{\pi}{9}$ then the amplitude of $Z_1 =$

① $\frac{7\pi}{36}$ ② $\frac{5\pi}{36}$

③ $\frac{\pi}{3}$ ④ $\frac{\pi}{4}$

$\theta_1 + \theta_2 = \frac{5\pi}{18}$ and $\theta_1 - \theta_2 = \frac{\pi}{9}$ $\therefore \theta_1 = 35^\circ = \frac{7\pi}{36}$

Q(4) If the number of terms in the expansion $(X + Y)^{2n-1}$ equals 12 terms then n=.....

① 5 ② 6

③ 7 ④ 8

$2n - 1 + 1 = 12 \quad \therefore 2n = 12 \quad \therefore n = 6$

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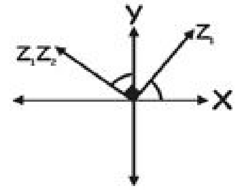
Q(5) In the opposite figure: If Z_1 and Z_2 and Z_1Z_2 are complex numbers then $Z_2 =$



① -2i ② -i
③ i ④ 2i

$$Z_2 = \frac{Z_1Z_2}{Z_1} = \frac{re^{\left(\frac{1}{2}\pi+\theta\right)}}{re^{\theta i}} = e^{\frac{1}{2}\pi i} = i$$

Q(6) The radius length of the sphere $X^2 + Y^2 + Z^2 - 2X - 6Y + 10Z - 1 = 0$ equals ....length unit

① 3 ② 4
③ 5 ④ 6

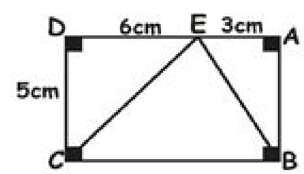
Center =(1,3,-5) radius $= \sqrt{1^2 + 3^2 + (-5)^2 + 1} = 6$

Q(7) If A=(2, -1,3) and B=(-2,2,-9) then the length of $\overline{AB}$ equals

① 12 ② 13
③ 14 ④ 15

$$AB = \sqrt{(2+2)^2 + (-1-2)^2 + (3+9)^2} = 13$$

Q(8) In the opposite figure ABCD is a rectangle $E \in \overline{AD}$ then $\overrightarrow{EB} \bullet \overrightarrow{EC} =$



① 7 ② 8
③ 9 ④ 10

$C = (0,0)$, $B = (9,0)$ $E = (6,5)$
$EB = B - E = (3,-5)$ $EC = C - E = (-6,-5)$
$EB \bullet EC = (3,-5) \bullet (-6,-5) = -18 + 25 = 7$

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Q(9)

If $\vec{A}=(2,3,-4)$ and $\vec{B}=(4,2,m)$ and $\vec{A} \perp \vec{B}$ then m=...

① 1 ② 2

③ 3 ④ $\frac{7}{2}$

$A \bullet B = 0 \quad \therefore 2 \times 4 + 3 \times 2 - 4m = 0 \quad \therefore 8 + 6 - 4m = 0$

$\therefore 14 - 4m = 0 \qquad \therefore m = \frac{7}{2}$

Q(10) Number of ways that a team of six members can be formed from eight girls and six boys such that the team must include exaltedly 3 boys

① 2110 ② 1120

③ 1008 ④ 810

${}^{8}C_{3} \times {}^{6}C_{3} = 1120$

Q(11) $\sqrt{5 + 12i} = \ldots\ldots$

① $\pm(2 + 3i)$ ② $\pm(3 + 2i)$

③ $\pm(2 - 3i)$ ④ $\pm(3 - 2i)$

$\sqrt{5 + 12i} = X + iY \quad \therefore 5 + 12i = X^2 - Y^2 + 2XYi \quad \therefore 2XYi = 12 \therefore XY = 6$

$\therefore Y = \frac{6}{X} \qquad \therefore X^2 - Y^2 = 5 \quad \therefore X^2 - \left(\frac{6}{X}\right)^2 = 5 \quad \therefore X^2 - \frac{36}{X^2} = 5$

$X^4 - 5X^2 - 36 = 0 \quad \therefore X = \pm 3 \quad , Y = \pm 2$

Q(12) If the side length of a triangle are $\frac{1}{2}\lfloor n$, $\lfloor n-2$ and $\lfloor 2-n$ cm

Then the numerical value of the area of the triangle =....cm^2

① $\sqrt{3}$ ② $\frac{\sqrt{3}}{4}$

③ $\frac{\sqrt{3}}{2}$ ④ $\frac{2\sqrt{3}}{3}$

$n - 2 \geq 0 \quad ,2 - n \geq 0 \quad \therefore n = 2 \text{ or } 3$

if $n = 2$ sides are 1, 1, 1 area $= \frac{1}{2} \times 1 \times 1 \times \sin 60^\circ = \frac{\sqrt{3}}{4}$

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Q(13) Find measure of the angle included between the straight line $L : \frac{X-3}{\sqrt{2}} = \frac{Y-1}{1} = \frac{-Z-2}{1}$ and the plane $\sqrt{2}X - Y - Z + 5 = 0$

$$\cos\theta = \frac{(\sqrt{2},1,1)\bullet(\sqrt{2},-1,-1)}{\sqrt{(\sqrt{2})^2+1^2+1^2}\times\sqrt{(\sqrt{2})^2+1^2+1^2}} = \frac{2-1-1}{} = 0$$

$\therefore \theta = 90^\circ$

Q(14) Find the standard form and the general form of the equation of the plane passing through point (3 , -5 , 2) and the vector n = (2 , 1, 1) is normal to the plane.

$\boxed{n\bullet r = n\bullet A}$

$(2,1,1)\bullet[(X,Y,Z)-(3,-5,2)] = 0 \quad \therefore (2,1,1)\bullet(X-3, Y+5, Z-2) = 0$

$2(X-3)+1(Y+5)+1(Z-2) = 0 \quad \therefore 2X-6+Y+5+Z-2 = 0$

$2X+Y+Z-3 = 0$

Q(15) Without expanding the determinant prove: $\begin{vmatrix}1 & 4 & 1\\ 0 & -1 & 2\\ 4 & 18 & 0\end{vmatrix} = 0$

$r_3 + 2r_2 \quad \therefore \begin{vmatrix}1 & 4 & 1\\ 0 & -1 & 2\\ 4 & 16 & 4\end{vmatrix}$ take 4 common factor from r_3

$\therefore 4\begin{vmatrix}1 & 4 & 1\\ 0 & -1 & 2\\ 4 & 16 & 4\end{vmatrix} = 4\begin{vmatrix}1 & 4 & 1\\ 0 & -1 & 2\\ 1 & 4 & 1\end{vmatrix}$

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Q(16)

Find the volume of the parallelepiped which three of its adjacent sides are represented by the vectors
$-12i - 3k$, $3j - K$ and $2i + j - 15K$

$$V = \begin{vmatrix} -12 & 0 & -3 \\ 0 & 3 & -1 \\ 2 & 1 & -15 \end{vmatrix} = 546$$

Q(17)Find the vector form of the equation of the straight line passing through point (3, -1, 0) and the vector (-2, 4 , 3) is a direction vector for it.

$\vec{r}$ = point + t (direction vector)

❶The vector equation:

$$\vec{r} = (3, -1, 0) + t(-2, 4, 3)$$

❷Parametric equations:

$$(X, Y, Z) = (3, -1, 0) + t(-2, 4, 3)$$

$\therefore X = 3 - 2t$, $Y = -1 + 4t$, $Z = 0 + 3t$

❸The Cartesian equation:

$\therefore t = \frac{X-3}{-2}$, $t = \frac{Y+1}{4}$, $t = \frac{Z}{3}$ $\therefore \frac{X-3}{-2} = \frac{Y+1}{4} = \frac{Z}{3}$

Q(18) Solve the following equations
X-Y+Z=2 , 2X+3Y-Z=5 , 3X-5Y+2Z=-1 using the multiplicative inverse of the matrix

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \frac{1}{11}\begin{pmatrix} -1 & 3 & 2 \\ 7 & 1 & -3 \\ 19 & -2 & -5 \end{pmatrix}\begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

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Q(19) If Z is a complex number , the amplitude of $(Z + i) = \frac{\pi}{4}$

And the amplitude of $(Z - 3) = \frac{3\pi}{4}$ find Z in the algebraic form

Let $Z = X + iY \quad \therefore (X + iY + i) = X + i(Y + 1)$

$\therefore \tan\theta = \frac{Y+1}{X} = \tan 45^\circ = 1 \quad \therefore X = Y + 1$

$\therefore (X + iY - 3) = (X - 3) + iY$

$\therefore \frac{Y}{X-3} = \tan 135^\circ = -1 \quad \therefore Y = -X + 3$

$\therefore Z = 2 + i$

Q(20) If the coefficients of the 4^{th} , 5^{th} and 6^{th} term respectively in the expansion of $(2X + Y)^n$ form an arithmetic sequence find the value of n

$2\,\text{coeff}.T_5 = \text{coeff}.T_4 + \text{coeff}.T_6 \quad \therefore \div T_5$

$\frac{\text{coeff}.T_4}{\text{coeff}.T_5} + \frac{\text{coeff}.T_6}{\text{coeff}.T_5} = 2$

$\frac{4}{n-4+1} \times 2 + \frac{n-5+1}{5} \times \frac{1}{2} = 2$

$\frac{8}{n-3} + \frac{n-4}{10} = 2$

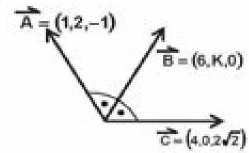
$\therefore n = 19 \quad \text{or } n = 8$

Algebra and solid Exam

Answer the following questions

From 1to 12 choose the correct answer

Q(1) In the opposite figure, the value of k = ...



① 9 ② 3

③ 6 ④ 4

$(6,K,0)\bullet(4,0,2\sqrt{2}) = (6,K,0)\bullet(1,2,-1)$

$24 = 6 + 2K \quad \therefore 2K = 18 \quad \therefore K = 9$

Q(2) $\left(1-\frac{1}{\omega}\right)\left(1-\frac{1}{\omega^2}\right)\left(1-\frac{1}{\omega^4}\right)\left(1-\frac{1}{\omega^8}\right)\ldots\ldots \times$ up to 10 factors

① 243 ② 0

③ 200 ④ 201

$= \left[(1-\omega^2)(1-\omega)\right]^5 = (1-\omega-\omega^2+\omega^3)^5 = (2+1)^5 = 243$

243

Q(3) The rank of the matrix $A = \begin{pmatrix} -1 & 0 & 0 \\ -1 & -1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$ equals

① 1 ② 0

③ 2 ④ 3

$\because |A| \neq 0$ rank is 3

Q(4) $1 - 6X + \frac{6\times5}{2\times1}X^2 - \frac{6\times5\times4}{3\times2\times1}X^3 + \ldots\ldots + X^6 = 64$ then X=.....

① -1 ② 2

③ 3 ④ {-1,3}

$(1-X)^6 = 64 \quad \therefore 1 - X = \pm 2 \quad \therefore X = \{-1,3\}$

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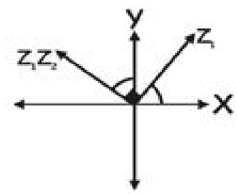
Q(5) In the opposite figure: If Z_1 and Z_2 and Z_1Z_2 are complex numbers then Z_2 =



① -2i ② -i

③ i ④ 2i

$Z_2 = \frac{Z_1Z_2}{Z_1} = \frac{re^{\left(\frac{1}{2}\pi+\theta\right)i}}{re^{\theta i}} = e^{\frac{1}{2}\pi i}$

Q(6) If A form an equal angles θ with the three axis and $\|A\| = 9$ then θ =…….

① $\cos^{-1}\frac{1}{\sqrt{2}}$ ② $\cos^{-1}\frac{1}{9}$

③ $\cos^{-1}\frac{1}{\sqrt{3}}$ ④ $\cos^{-1}\frac{1}{3}$

$3\cos^2\theta = 1 \therefore \cos\theta = \frac{1}{\sqrt{3}}$

Q(7) If $(a+b)\bullet(a-b) = 63$ and $|a| = 8|b|$ then $|a|$ =

① 8 ② 64

③ 16 ④ 4

$a^2 - b^2 = 63 \quad \therefore a^2 - \frac{1}{64}a^2 = 63 \quad \therefore a^2 = 64 \quad \therefore |a| = 8$

Q(8) The direction cosine of the line
$X = 3 - 2Y \; , \; Z = 2Y - 1$ is …….

① $\frac{-2}{3},\frac{1}{3},\frac{2}{3}$ ② $\frac{-2}{3},\frac{-1}{3},\frac{2}{3}$

③ $\frac{2}{3},\frac{1}{3},\frac{2}{3}$ ④ $\frac{-2}{3},-\frac{1}{3},\frac{2}{3}$

$\frac{X-3}{-2} = \frac{Y}{1} = \frac{Z+1}{2}$

Algebra and solid Exam

Q(9) If A(7,-1,,8) and B(11,2,-4) then the length of $\overline{AB}$ =

① 10 ② 11

③ 12 ④ 13

$AB\sqrt{(11-7)^2+(2+1)^2+(-4-8)^2}=13$

Q(10) From a class of 9 boys and 6 girls it's required to select a 4-person team of the same gender.

① 3384 ② 143

③ 145 ④ 141

${}^9C_4 + {}^6C_4 = 126 + 15 = 141$

Q(11) If $(1+\omega^2)^n = (1+\omega)^n$ then the least value of the positive integer n equals

① 2 ② 3

③ 6 ④ 5

$(-\omega)^6 = (-\omega^2)^6 \quad \therefore n = 6$

Q(12) If $\frac{a^2+b^2}{a+bi} = 2+3i$ then a × b = ... where a and b ∈ R

① -6 ② 5

③ -5 ④ 6

$\frac{a^2+b^2}{a+bi} \times \frac{a-bi}{a-bi} = \frac{(a^2+b^2)(a-bi)}{(a^2+b^2)} = 2+3i \quad \therefore a - bi = 2+3i \therefore a \times b = -6$