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Algebra and solid Exam

Answer the following questions 20 questions

From 1to 12 choose the correct answer

Q(1)If the plane $10X + 12Y + 6Z = 60$ intersected with the
and Z at the points A ,B and C respectively then the volum
ABCO where O is the origin point equals .....cube unit

① 20 ② 30

③ 50 ④ other wises

Volume of pyramid $= \frac{1}{3}$ base area × height

$= \frac{1}{3}\left(\frac{1}{2} \times 5 \times 6\right) \times 10 = 50$

Q(2) In the opposite figure: Z_1, Z_2 are two complex numbers th

① 2 ② 2i

③ -2 ④ -2i

$$\frac{8(\cos 180° - \theta + i\sin 180° - \theta)}{4(\cos -90° - \theta + i\sin -90° - \theta)} = 2(\cos 270° + i\sin 270°) = -2i$$

Q(3) If Z_1, Z_2 are two complex numbers the amplitude of $(Z_1Z_2$

And the amplitude of $\left(\frac{Z_1}{Z_2}\right) = \frac{\pi}{9}$ then the amplitude of $Z_1 =$

① $\frac{7\pi}{36}$ ② $\frac{5\pi}{36}$

③ $\frac{\pi}{3}$ ④ $\frac{\pi}{4}$

$\theta_1 + \theta_2 = \frac{5\pi}{18}$ and $\theta_1 - \theta_2 = \frac{\pi}{9}$ $\therefore \theta_1 = 35° = \frac{7\pi}{36}$

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Q(5) In the opposite figure: If Z_1 and Z_2 and Z_1Z_2 are comple numbers then Z_2 =

① -2i ② -i

③ i ④ 2i

Z_1Z_2

$$Z_2 = \frac{Z_1Z_2}{Z_1} = \frac{re^{\left(\frac{1}{2}\pi+\theta\right)}}{re^{\theta i}} = e^{\frac{1}{2}\pi i} = i$$

Q(6) The radius length of the sphere $X^2 + Y^2 + Z^2 - 2X - 6Y + 10Z - 1 = 0$ equals ..... lengt

① 3 ② 4

③ 5 ④ 6

Center =(1,3,-5) radius $= \sqrt{1^2 + 3^2 + (-5)^2 + 1} = 6$

Q(7) If A=(2, -1,3) and B=(-2,2,-9) then the length of $\overline{AB}$ equ

① 12 ② 13

③ 14 ④ 15

$$AB = \sqrt{(2+2)^2 + (-1-2)^2 + (3+9)^2} = 13$$

Q(8) In the opposite figure ABCD is a rectangle $E \in \overline{AD}$ then $\overrightarrow{EB} \bullet \overrightarrow{EC} =$

D 6c

① 7 ② 8

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Q(9)

If $\vec{A}=(2,3,-4)$ and $\vec{B}=(4,2,m)$ and $\vec{A} \perp \vec{B}$ then m=...

① 1 ② 2

③ 3 ④ $\frac{7}{2}$

$A \bullet B = 0 \therefore 2 \times 4 + 3 \times 2 - 4m = 0 \therefore 8 + 6 - 4m = 0$

$\therefore 14 - 4m = 0 \quad \therefore m = \frac{7}{2}$

Q(10) Number of ways that a team of six members can be forme
girls and six boys such that the team must include exaltedly 3 boy

① 2110 ② 1120

③ 1008 ④ 810

${}^{8}C_3 \times {}^{6}C_3 = 1120$

Q(11) $\sqrt{5+12i} =$.....

① $\pm(2+3i)$ ② $\pm(3+2i)$

③ $\pm(2-3i)$ ④ $\pm(3-2i)$

$\sqrt{5+12i} = X + iY \therefore 5 + 12i = X^2 - Y^2 + 2XYi \therefore 2XYi = 12 \therefore$

$\therefore Y = \frac{6}{X} \quad \therefore X^2 - Y^2 = 5 \quad \therefore X^2 - \left(\frac{6}{X}\right)^2 = 5 \quad \therefore X^2 - \frac{36}{X^2} = 5$

$X^4 - 5X^2 - 36 = 0 \quad \therefore X = \pm 3 \quad , Y = \pm 2$

Q(12) If the side length of a triangle are $\frac{1}{2}\lfloor n$, $\lfloor n-2$ and $\lfloor$
Then the numerical value of the area of the triangle =....cm²

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Q(13) Find measure of the angle included between the s

$L : \frac{X-3}{\sqrt{2}} = \frac{Y-1}{1} = \frac{-Z-2}{1}$ and the plane $\sqrt{2}X - Y - Z$

$$\cos\theta = \frac{(\sqrt{2},1,1)\bullet(\sqrt{2},-1,-1)}{\sqrt{(\sqrt{2})^2+1^2+1^2}\times\sqrt{(\sqrt{2})^2+1^2+1^2}} = \frac{2-1-1}{} = 0$$

$\therefore \theta = 90^\circ$

Q(14) Find the standard form and the general form of the
the plane passing through point (3 , -5 , 2) and the vector
n = (2 , 1, 1) is normal to the plane.

$\boxed{n \bullet r = n \bullet A}$

$(2,1,1)\bullet[(X,Y,Z)-(3,-5,2)] = 0 \quad \therefore (2,1,1)\bullet(X-3, Y+5, Z-2$

$2(X-3)+1(Y+5)+1(Z-2) = 0 \quad \therefore 2X - 6 + Y + 5 + Z - 2 = 0$

$2X + Y + Z - 3 = 0$

Q(15) Without expanding the determinant prove: $\begin{vmatrix} 1 & 4 & \\ 0 & -1 & \\ 4 & 18 & \end{vmatrix}$

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Q(16)

Find the volume of the parallelepiped which three of it adjacent sides are represented by the vectors $-12i - 3k$, $3j - K$ and $2i + j - 15K$

$$V = \begin{vmatrix} -12 & 0 & -3 \\ 0 & 3 & -1 \\ 2 & 1 & -15 \end{vmatrix} = 546$$

Q(17)Find the vector form of the equation of the straight line pa point (3, -1, 0) and the vector (-2, 4 , 3) is a direction vector for it.

$$\boxed{\vec{r} = \text{point} + t\ (\text{direction vector})}$$

❶ The vector equation:

$$\vec{r} = (3, -1, 0) + t(-2, 4, 3)$$

❷ Parametric equations:

$$(X, Y, Z) = (3, -1, 0) + t(-2, 4, 3)$$

$$\therefore X = 3 - 2t \quad , \quad Y = -1 + 4t \quad , \quad Z = 0 + 3t$$

❸ The Cartesian equation:

$$\therefore t = \frac{X-3}{-2} \quad , \quad t = \frac{Y+1}{4} \quad , \quad t = \frac{Z}{3} \qquad \therefore \frac{X-3}{-2} = \frac{Y+}{4}$$

Q(18) Solve the following equations

X-Y+Z=2 , 2X+3Y-Z=5 , 3X-5Y+2Z=-1 using the multiplicat the matrix

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Q(19) If Z is a complex number , the amplitude of (Z -
And the amplitude of $(Z - 3) = \frac{3\pi}{4}$ find Z in the algebr

Let $Z = X + iY \quad \therefore (X + iY + i) = X + i(Y + 1)$

$\therefore \tan\theta = \frac{Y+1}{X} = \tan 45^\circ = 1 \quad \therefore X = Y + 1$

$\therefore (X + iY - 3) = (X - 3) + iY$

$\therefore \frac{Y}{X-3} = \tan 135^\circ = -1 \quad \therefore Y = -X + 3$

$\therefore Z = 2 + i$

Q(20) If the coefficients of the 4th , 5th and 6th term respec
expansion of $(2X + Y)^n$ form an arithmetic sequence find t

$2\underline{coeff.}T_5 = \underline{coeff.}T_4 + \underline{coeff.}T_6 \quad \therefore \div T_5$

$\frac{\underline{coeff.}T_4}{\underline{coeff.}T_5} + \frac{\underline{coeff.}T_6}{\underline{coeff.}T_5} = 2$

$\frac{4}{n-4+1} \times 2 + \frac{n-5+1}{5} \times \frac{1}{2} = 2$

$\frac{8}{n-3} + \frac{n-4}{10} = 2$

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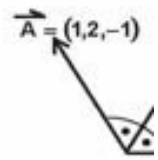
Answer the following questions

From 1to 12 choose the correct answer

Q(1) In the opposite figure, the value of k = ...



① 9 ② 3

③ 6 ④ 4

$(6,K,0)\bullet(4,0,2\sqrt{2}) = (6,K,0)\bullet(1,2,-1)$

$24 = 6 + 2K \quad \therefore 2K = 18 \quad \therefore K = 9$

Q(2) $\left(1-\frac{1}{\omega}\right)\left(1-\frac{1}{\omega^2}\right)\left(1-\frac{1}{\omega^4}\right)\left(1-\frac{1}{\omega^8}\right)\ldots\ldots\times$ up to 10 fc

① 243 ② 0

③ 200 ④ 201

$= \left[(1-\omega^2)(1-\omega)\right]^5 = (1-\omega-\omega^2+\omega^3)^5 = (2+1)^5 = 243$

243

Q(3) The rank of the matrix $A = \begin{pmatrix} -1 & 0 & 0 \\ -1 & -1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$ equals

① 1 ② 0

③ 2 ④ 3

$\because |A| \neq 0$ rank is 3

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Q(5) In the opposite figure: If Z_1 and Z_2 and Z_1Z_2 are com numbers then Z_2 =

① $-2i$ ② $-i$

③ i ④ $2i$

$Z_2 = \frac{Z_1Z_2}{Z_1} = \frac{re^{\left(\frac{1}{2}\pi+\theta\right)i}}{re^{\theta i}} = e^{\frac{1}{2}\pi i}$

Q(6) If A form an equal angles θ with the three axis ar then θ =.....

① $\cos^{-1}\frac{1}{\sqrt{2}}$ ② $\cos^{-1}\frac{1}{9}$

③ $\cos^{-1}\frac{1}{\sqrt{3}}$ ④ $\cos^{-1}\frac{1}{3}$

$3\cos^2\theta = 1 \therefore \cos\theta = \frac{1}{\sqrt{3}}$

Q(7) If $(a+b)\bullet(a-b) = 63$ and $|a| = 8|b|$ then $|a|$ =

① 8 ② 64

③ 16 ④ 4

$a^2 - b^2 = 63 \quad \therefore a^2 - \frac{1}{64}a^2 = 63 \quad \therefore a^2 = 64 \quad \therefore |a| = 8$

Q(8) The direction cosine of the line
$X = 3 - 2Y$, $Z = 2Y - 1$ is .....

① $\frac{-2}{}\ \frac{1}{}\ \frac{2}{}$ ② $\frac{-2}{}\ \frac{-1}{}\ \frac{2}{}$

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Q(9) If A(7,-1,,8) and B(11,2,-4) then the length of $\overline{AB}$ =

① 10 ② 11

③ 12 ④ 13

$AB\sqrt{(11-7)^2+(2+1)^2+(-4-8)^2}=13$

Q(10) From a class of 9 boys and 6 girls it's required to s
4-person team of the same gender.

① 3384 ② 143

③ 145 ④ 141

${}^9C_4 + {}^6C_4 = 126 + 15 = 141$

Q(11) If $(1+\omega^2)^n = (1+\omega)^n$ then the least value of the posi
equals

① 2 ② 3

③ 6 ④ 5

$(-\omega)^6 = (-\omega^2)^6 \quad \therefore n = 6$

Q(12) If $\frac{a^2+b^2}{a+bi} = 2+3i$ then $a \times b = \ldots\ldots$ where a and